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Nontrivial linear effects of dispersion at the interaction point

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Effects of a large dispersion at the interaction point are studied within the linear approximation. Several effects exist on the synchrotron motion, including the synchrotron tune shift, the bunch lengthening, and energy spread modification, which might lead to instability, luminosity decrease, and an increase of the collision energy resolution. [S1063-651X(99)50801-5]

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I. INTRODUCTION

In the conventional colliders, the dispersion at the interaction point (IP) is designed to be zero, and might have a small value due to machine errors. The effects of such a small dispersion at the IP have been studied regarding the dispersion as a small perturbation [1,2]. Often, the synchrotron degree of freedom was treated as a large heat bath which is not affected at all [3]. Such a treatment might be reasonable when the dispersion is small. Recently, however, the monochromatization has been considered seriously for future τ-charm factories [4], where a rather large dispersion exists at the IP with opposite signs for both beams. In this case, the dispersion effects can no longer be discussed in the perturbative sense.

In e^+e^- storage rings, the synchrotron oscillation always exists. In standard textbooks such as Ref. [5], however, the dispersion is defined under the assumption that the energy of an electron can be considered as a constant. This is misleading in the presence of the synchrotron oscillation [6]. This approach appears to be intuitively valid when the absolute value of the synchrotron tune ν_z is very small, but we will see that even this is not true.

To see the nonperturbative effects with large dispersion, a weak-strong simulation has been done on the basis of the three-dimensional symplectic beam-beam mapping [7], which showed the satisfactory performance of this scheme for the Beijing Tau-Charm factory [8]. On the other hand, with simulation only, it is difficult to understand the general properties of such a scheme.

The aim of this Rapid Communication is to discuss the effects of the dispersion at the IP, paying enough attention to the mutual interaction between the betatron and the synchrotron degrees of freedom, and to study the possible problems associated with the monochromatization within the linear approximation of the beam-beam force. Considering the role of such approximation in the usual beam-beam study, we can expect a good insight into these effects. It appears to be the most basic approach in studying the dispersion effects but has not yet been investigated carefully enough [9].

We first discuss linear symplectic dynamics in the next section. In Sec. III, we study problems associated with beam sizes which are determined by the stochastic effects due to the synchrotron radiation. Section IV will be devoted to discussions and conclusions.

II. SYMPLECTIC EFFECTS

For simplicity, we consider the synchrotron motion and one betatron oscillation degree of freedom only. The latter is called ‘‘vertical’’ but it can be horizontal as well. Let us define the physical variables of a particle for the betatron and synchrotron motions: $\mathbf{x}=(y,p_y,z,\epsilon)$, where y is the vertical coordinate, p_y is the vertical momentum normalized by the (constant) momentum p_0 of the reference particle, z is the time advance relative to the reference particle multiplied by the light velocity c, and $\varepsilon=(E-E_0)/E_0$ is the energy deviation from the nominal value E_0 and normalized by it.

The one turn matrix from IP ($s=0$) to IP (excluding the beam-beam kick) can be put in the following form [10]:

$$M_{\rm arc}=M(0_-,0_+)=H_0B_0\hat{M}_{\rm arc}B_0^{-1}H_0^{-1}, \tag{1}$$

where $\hat{M}_{\rm arc}=\mathrm{diag}(r(\mu_y^0),r(\mu_z^0))$, $B_0=\mathrm{diag}(b_y^0,b_z^0)$, with

$$r(\mu_{y,z}^0)=\begin{pmatrix} \cos\mu_{y,z}^0 & \sin\mu_{y,z}^0 \\ -\sin\mu_{y,z}^0 & \cos\mu_{y,z}^0 \end{pmatrix}, \tag{2}$$

$$b_{y,z}^0=\mathrm{diag}(\sqrt{\beta_{y,z}^0},1/\sqrt{\beta_{y,z}^0}),$$
$$H_0=\begin{pmatrix} I & h_0 \\ h_0 & I \end{pmatrix},\quad h_0=\begin{pmatrix} 0 & D_0 \\ 0 & 0 \end{pmatrix}. \tag{3}$$

$\mu^0=2\pi\nu^0$, ν^0 being the nominal tune, $\beta_{y,z}^0$ the nominal betatron functions at IP ($\beta_z^0\equiv\sigma_z^0/\sigma_\varepsilon^0$, where σ_z^0 and σ_ε^0 being the nominal bunch length and energy spread, respectively), and D_0 the dispersion at IP. Note that H_0, B_0, and $\hat{M}_{\rm arc}$ are symplectic. The nominal synchrotron tune ν_z^0 is negative for conventional electron machines with positive momentum compaction factor α_p. We shall, however, consider both signs for ν_z^0 because the negative α_p [11] option is being considered, which makes ν_z^0 positive. We have assumed that there is only one IP that is a symmetric point with respect to betatron and synchrotron motions. We have also implicitly assumed that dispersion does not exist in cavities.

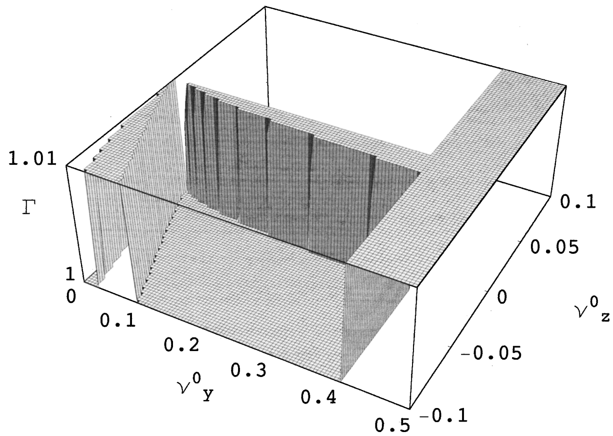

FIG. 1. Growth rate Γ as a function of tunes with $(\xi_0, D_0) = (0.05, 0.4\,\mathrm{m})$. Three unstable regions can be seen.

Turning on the beam-beam kick at IP, the complete one-turn map is

$$M = M_{bb}^{1/2} M_{\mathrm{arc}} M_{bb}^{1/2}, \quad M_{bb} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ -4\pi\xi_0/\beta_y^0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{4}$$

with ξ_0 being the vertical (nominal) beam-beam parameter.

We can get the perturbed tunes $\nu = \mu/2\pi$ easily: the eigenvalues of M are $\exp i\mu_\pm$, where

$$2\cos\mu_\pm = \cos\mu_y^0 + \cos\mu_z^0 - 2\pi\xi_0 \sin\mu_y^0 - 2\pi\xi_0\chi \times \sin\mu_z^0 \pm \sqrt{d}, \tag{5}$$

$$d = [\cos\mu_y^0 - \cos\mu_z^0 - 2\pi\xi_0(\sin\mu_y^0 - \chi\sin\mu_z^0)]^2 + 16\pi^2\xi_0^2\chi\sin\mu_y^0\sin\mu_z^0, \tag{6}$$

$\chi = D_0^2/\beta_y^0\beta_z^0 = D_0^2\sigma_\varepsilon^0/\beta_y^0\sigma_z^0$ being the synchrotron tune shift factor. To lowest order in ξ_0, we get

$$\nu_y^0 \to \nu_y^0 + \xi_0, \quad \nu_z^0 \to \nu_z^0 + \xi_0\chi. \tag{7}$$

The first of Eq. (7) is the well-known betatron tune shift, while the second is a synchrotron tune shift. Equation (5) implies that the system becomes unstable when (i) $\nu_y^0 \lesssim$ half integers (betatron instability), (ii) $\nu_z^0 \lesssim$ half integers (synchrotron instability), (iii) $\nu_z^0 + \nu_y^0 \lesssim$ integers (synchro-betatron instability). The first two correspond to the case where $|\cos\mu| > 1$, while the last is associated with the case where $d < 0$.

The instability regions in the (ν_y^0, ν_z^0) plane are shown in Fig. 1 in terms of the growth rate Γ, the largest eigenvalue of M in absolute value. The three unstable regions stated above are clearly seen. The unstable regions become wider for larger values of ξ_0 and D_0. As seen from the figure, a machine might be intrinsically more stable when $\nu_z^0 > 0$, because we can get rid of the synchrotron and synchro-betatron instabilities. In Fig. 1 (and hereafter), the model parameters listed in Table I were used, unless otherwise specified.

TABLE I. Standard model parameters used in this paper ($\chi = 0.2$).

D_0	0.4 m	ξ_0	0.05
β_y^0	0.03 m	β_z^0	26.3 m
ϵ_y^0	4×10^{-9} m	ϵ_z^0	3.8×10^{-6} m
σ_ε^0	3.8×10^{-4}	σ_z^0	0.01 m
ν_y^0	0.1	ν_z^0	-0.08
T_y	1000	T_z	500

It may be useful to note that the synchrotron tune shift effect is remarkable for (i) large D_0 (ii) large σ_ε^0, (iii) small σ_z^0, (iv) small β_y^0, and (v) small $|\nu_z^0|$. Items (iii), (iv), and (v) are general design trends when we want to have large luminosity by making the beam size small and avoiding synchro-betatron sidebands [12]. The condition $\chi \ll 1$ is equivalent to $(D_0\sigma_\varepsilon^0)^2/\beta_y^0 \ll \epsilon_z^0$. On the other hand, for the monochromatization to be useful, the left-hand side should be much larger than the vertical emittance so that the parameters should satisfy

$$\epsilon_y^0 \ll (D_0\sigma_\varepsilon^0)^2/\beta_y^0 \ll \epsilon_z^0. \tag{8}$$

Note that a naive guess that the energy can be well approximated as a constant (coasting beam) when $|\nu_z^0| \sim 0$ is totally wrong in this case. In fact, when $\nu_z^0 \lesssim 0$, the motion becomes unstable by a tiny perturbation due to the beam-beam interaction: $\nu_z^0 = 0$ is a singular point and the coasting beam approximation is dangerous in this case.

Let us briefly discuss the coherent motion for the strong-strong case. The rigid Gaussian model [13] can be applied. The π mode exists which consists of the variable

$$\mathbf{x}_\pi = (\overline{y^+} - \overline{y^-}, \overline{p_y^+} - \overline{p_y^-}, \overline{z^+} + \overline{z^-}, \overline{\epsilon^+} + \overline{\epsilon^-}).$$

Here, $\overline{x_i}$ stands for $\langle x_i \rangle$, where $\langle\,\rangle$ is the average over all the particles and $\pm$ refers to the $e^\pm$ beam. This mode shows the same instability structure as the single particle case discussed above. The other mode is indifferent to the beam-beam interaction.

III. RADIATION EFFECTS

Let us discuss the equilibrium value of the second-order moments,

$$\sigma_{ij} = \langle (x_i - \overline{x_i})(x_j - \overline{x_j}) \rangle. \tag{9}$$

If radiation is included, the equilibrium value of σ is determined by the following equation [14]:

$$\sigma = M_{bb}^{1/2}[\overline{\Lambda} M_{\mathrm{arc}} \sigma (\overline{\Lambda} M_{\mathrm{arc}})^t + (I - \overline{\Lambda}^2)\overline{E}](M_{bb}^t)^{1/2}, \tag{10}$$

where $M_{\mathrm{arc}} = H_0 B_0 \hat{M}_{\mathrm{arc}} (H_0 B_0)^{-1}$,

$$\overline{\Lambda} = H_0 B_0 \Lambda (H_0 B_0)^{-1}, \quad \overline{E} = H_0 B_0 E (H_0 B_0)^t, \tag{11}$$

$$\Lambda = \mathrm{diag}(\lambda_y, \lambda_y, 1, \lambda_z^2), \quad E = \mathrm{diag}(\epsilon_y^0, \epsilon_y^0, \epsilon_z^0, \epsilon_z^0). \tag{12}$$

In Fig. 2 we show the diagonal terms (σ_{ii}) of the envelope matrix as functions of ξ_0 for $\nu_z^0 = 0.08$. A rapid increase with

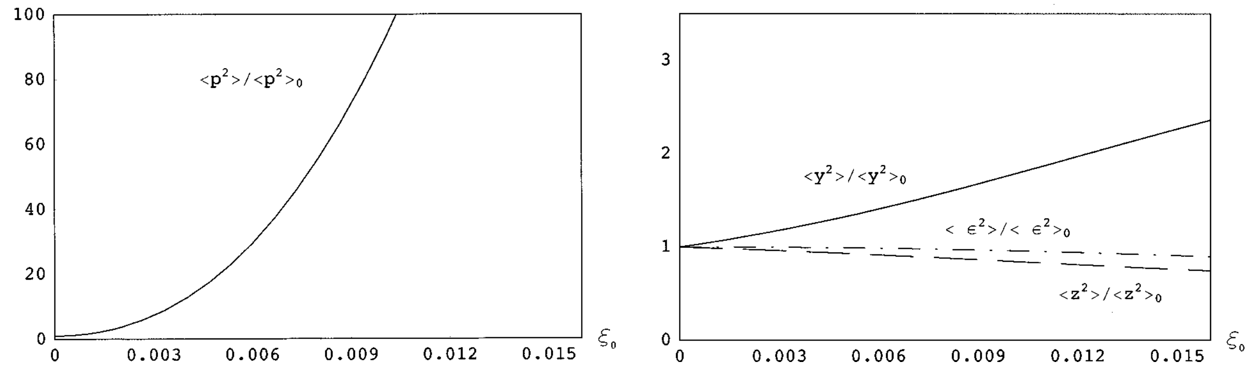

FIG. 2. rms beam sizes as functions of ξ_0. Left: $\langle p^2\rangle/\langle p^2\rangle_0$ (solid line). Right: $\langle y^2\rangle/\langle y^2\rangle_0$ (solid line), $\langle z^2\rangle/\langle z^2\rangle_0$ (dashed line), and $\langle \varepsilon^2\rangle/\langle \varepsilon^2\rangle_0$ (dashed-dotted line) for $\nu_y^0=0.05$, $\nu_z^0=0.08$. The index 0 refers to $\xi_0=0$. These parameters give no instability.

ξ_0 is observed for σ_{11} and in particular for $\sigma_{22}=\langle p_y^2\rangle$. They increase in a similar manner regardless of the sign of ν_z^0. For $\nu_z^0=-0.08$, the increase might be easier to understand, because the threshold of the instability is close ($\xi_0=0.0158$).

In approximating the beam-beam kick by a single kick, we have assumed that the bunch length is small enough. Let us examine if this assumption is self-consistent after the beam-beam kick is turned on. We define the effective betatron function as $\beta_{\rm eff}=\sqrt{\sigma_{11}/\sigma_{22}}$. When $\xi_0\simeq 0$, we get $\beta_{\rm eff}\simeq\beta_y^0\sqrt{1+D_0^2(\sigma_\epsilon^0)^2/\beta_y^0\epsilon_y^0}$. Let us define the hourglass ratio as $R_h=\beta_{\rm eff}/\sigma_z$. When $R_h\lesssim 1$, the hourglass effects (luminosity degradation [15] and possible introduction of new synchro-betatron coupling) become serious and the single kick approximation is no longer valid [7,16].

In Fig. 3, we show R_h for different values of ν_z^0. A rapid decrease of R_h with ξ_0 can be seen. For the present model parameters, R_h is still larger than unity and the single kick approximation is valid. Considering that the effect is remarkable, one should pay enough attention to this effect in deciding machine parameters.

The very purpose of monochromatization is to make the spread σ_w of the collision energy $w\equiv\varepsilon_++\varepsilon_-$ much less than the nominal one ($\sqrt{2}\sigma_\epsilon^0$). Thus, $\sqrt{2}\sigma_\epsilon^0/\sigma_w$ is as important as the luminosity L. It measures the effectiveness of the monochromatization.

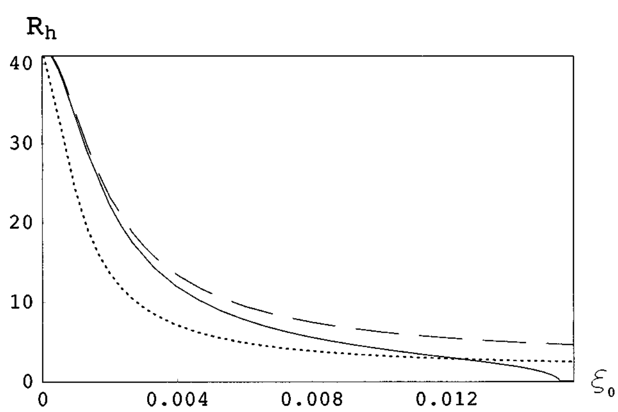

FIG. 3. The hourglass ratio $R_h(\xi_0)$ with $\nu_y^0=0.05$ and different values of ν_z^0: $\nu_z^0=\pm 0.03$ (dotted line), $\nu_z^0=-0.08$ (solid line), $\nu_z^0=0.08$ (dashed line). $R_h(0)=41$. Note that for $\nu_z^0=-0.08$ the instability threshold is at $\xi_0=0.0158$.

The luminosity density [17] with respect to w is proportional to

$$\Lambda(w)=\int f_+(y,\varepsilon_+)f(y,\varepsilon_-)\,\delta(w-\varepsilon_+-\varepsilon_-)\,dy\,d\varepsilon_+d\varepsilon_-, \tag{13}$$

where f is the projection of the phase space distribution function into the subspace (y,ε). The σ_w can be computed as $\sigma_w^2=\int w^2\Lambda(w)dw/\int\Lambda(w)dw$, and without the beam-beam effect it is equal to $\sigma_w^0=\sqrt{2}\sigma_\epsilon^0/[1+(D_0\sigma_\epsilon^0)^2/(\epsilon_y^0\beta_y^0)]^{1/2}$.

If we assume that the two beams are affected symmetrically, i.e., $\sigma_{11}^+=\sigma_{11}^-$, $\sigma_{44}^+=\sigma_{44}^-$, and $\sigma_{14}^+=-\sigma_{14}^-$, where $\sigma_{ij}^\pm$ is σ_{ij} for $e^\pm$ beams, we get $\sigma_w=\sqrt{2(\sigma_{11}\sigma_{44}-\sigma_{14}^2)/\sigma_{11}}$. In Fig. 4 we show σ_w as a function of ξ_0. Note that σ_w approaches σ_ε^0 quickly with increasing ξ_0, thus making monochromatization less effective or even useless. It can be avoided when $0\lesssim\nu_y^0<|\nu_z^0|$ holds, which is a little difficult to achieve. This effect gives more stringent limit for the maximum value of ξ_0 than the single particle instability threshold.

IV. CONCLUSION

Through the dispersion at IP, the synchrotron and betatron motions influence each other, giving several nontrivial strong

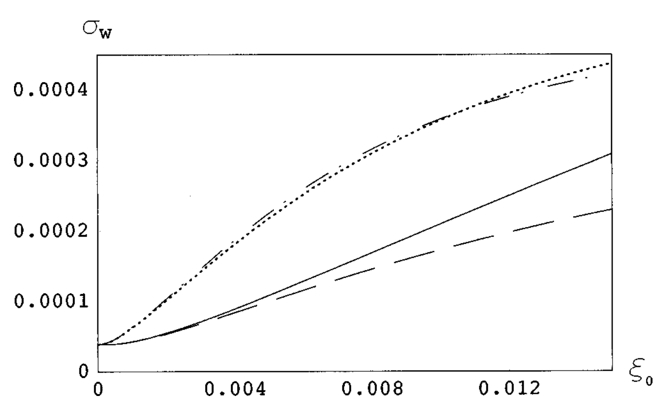

FIG. 4. Energy resolution σ_w vs ξ_0, with $\nu_y^0=0.05$ and different values of ν_z^0: $\nu_z^0=0.03$ (dash-dotted line), $\nu_z^0=-0.03$ (dotted line), $\nu_z^0=-0.08$ (solid line), and $\nu_z^0=0.08$ (dashed line). The nominal energy resolution is $\sigma_w^0=3.4\times 10^{-5}$. The real value can become comparable or even larger than the nominal energy spread $\sigma_\varepsilon^0=3.8\times 10^{-4}$.

effects on the synchrotron motion in addition to well-known transverse effects for rather small values of ξ_0.

They might limit the attainable value of ξ_0, or, equivalently, might impair the monochromatization scheme itself.

In this Rapid Communication the stress was on the treatment of the coupling between synchrotron and betatron motions in the symplectic way, while we used a very simple modeling of the beam, linear beam-beam force, very short bunch and so on. Even such a simple analysis predicts several nontrivial and dangerous effects overlooked before.

Within the present analysis, it seems difficult to avoid such dangerous effects with reasonable parameters. It might, however, come from the oversimplification of the model. Naturally enough, we need more detailed analysis. The present analysis may serve as a starting point and as a warning to simple minded approaches. More detailed and extended study will be published elsewhere.

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